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Chapter 6

Put-Call Parity and Synthetics

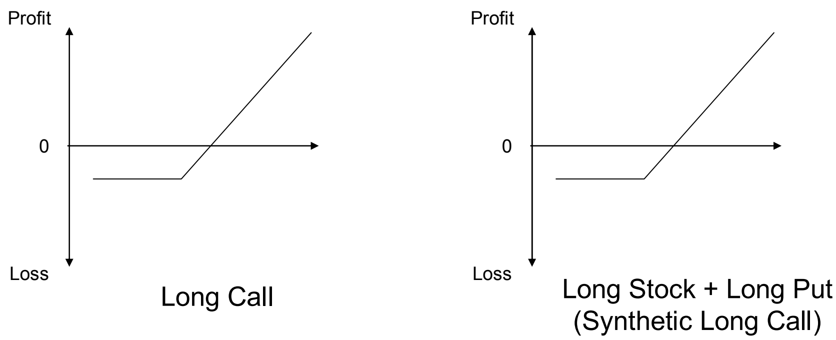
Excerpt from *Trading Option Greeks,* by Dan Passarelli

In order to understand more-complex spread strategies involving two or more options, it is essential to understand the arbitrage relationship of the put-call pair. Puts and calls of the same month and strike on the same underlying have prices that are defined in a mathematical relationship. They also have distinctly related vegas, gammas, thetas and deltas. This chapter will show how the metrics of these options are interrelated. It will also explore synthetics and the idea that by adding stock to a position, a trader may trade with indifference either a call or a put to the same effect.

Before the creation of the Black-Scholes model, option pricing was hardly an exact science. Traders had only a few mathematical tools available to compare the relative prices of options. One such tool, put-call parity, stems from the fact that puts and calls on the same class sharing the same month and strike can have the same functionality when stock is introduced.

For example, traders wanting to own a stock with limited risk can buy a married put: long stock and a long put on a share-for-share basis. The traders have infinite profit potential, and the risk of the position is limited below the strike price of the option. Conceptually, long calls have the same risk/reward profile—unlimited profit potential and limited risk below the strike. **FIGURE 6.1** is an overview of the at-expiration diagrams of a married put and a long call.

Figure 6.1 Long Call vs. Long Stock + Long Put (Married Put)



Married puts and long calls sharing the same month and strike on the same security have at-expiration diagrams with the same shape. They have the same volatility value and should trade around the same implied volatility. Strategically, these two positions provide the same service to a trader, but depending on margin requirements, the married put may require more capital to establish, because the trader must buy not just the option but also the stock.

The stock component of the married put could be purchased on margin. Buying stock on margin is borrowing capital to finance a stock purchase. This means the trader has to pay interest on these borrowed funds. Even if the stock is purchased without borrowing, there is opportunity cost associated with the cash used to pay for the stock. The capital is tied up. If the trader wants to use funds to buy another asset, he will have to borrow money, which will incur an interest obligation. Furthermore, if the trader doesn't invest capital in the stock, the capital will rest in an interest-bearing account. The trader foregoes that interest when he buys a stock. However the trader finances the purchase, there is an interest cost associated with the transaction.

Both of these positions, the long call and the married put, give a trader exposure to stock-price advances above the strike price. The important difference between the two trades is the value of the stock below the strike price—the part of the trade that is not at risk in either the long call or the married put. On this portion of the invested capital, the trader pays interest with the married put (whether actually or in the form of opportunity cost). This interest component is a pricing consideration that adds cost to the married put and not the long call.

So if the married put is a more expensive endeavor than the long call because of the interest paid on the investment portion that is below the strike, why would anyone buy a married put? Wouldn't traders instead buy the less expensive—less capital-intensive—long call? Given the additional interest expense, they would rather buy the call. This relates to the concept of arbitrage. Given two effectively identical choices, rational traders will choose to buy the less expensive alternative. The market as a whole would buy the calls, creating demand which would cause upward price pressure on the call. The price of the call would rise until its interest advantage over the married put was gone. In a robust market with many savvy traders, arbitrage opportunities don't exist for very long.

It is possible to mathematically state the equilibrium point toward which the market forces the prices of call and put options by use of the put-call parity. As shown in Chapter 2, the put-call parity states

$$c + PV(x) = p + s$$

where c is the call premium, PV(x) is the present value of the strike price, p is the put premium, and s is the stock price.

Another, less academic and more trader-friendly way of stating this equation is

$$\text{Call} + \text{Strike} - \text{Interest} = \text{Put} + \text{Stock}$$

where Interest is calculated as

Interest = Strike x Interest Rate x (Days to Expiration/365)[1]

The two versions of the put-call parity stated here hold true for European options on non–dividend-paying stocks.

Dividends

Another difference between call and married-put values is dividends. A call option does not extend to its owner the right to receive a dividend payment. Traders, however, who are long a put and long stock are entitled to a dividend if it is the corporation’s policy to distribute dividends to its shareholders.

An adjustment must be made to the put-call parity to account for the possibility of a dividend payment. The equation must be adjusted to account for the absence of dividends paid to call holders. For a dividend-paying stock, the put-call parity states

Call + Strike – Interest + Dividend = Put + Stock

The interest advantage and dividend disadvantage of owning a call is removed from the market by arbitrageurs. Ultimately, that is what is expressed in the put-call parity. It’s a way to measure the point at which the arbitrage opportunity ceases to exist. When interest and dividends are factored in, a long call is an equal position to a long put paired with long stock. In options nomenclature, a long put with long stock is a synthetic long call. Algebraically rearranging the above equation:

Call = Put + Stock – Strike + Interest – Dividend

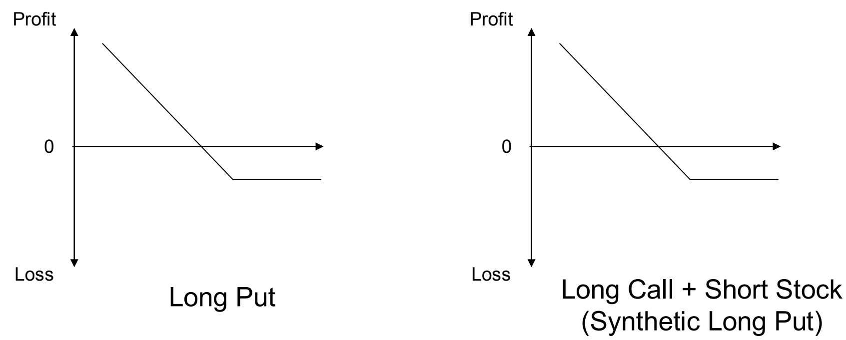
The interest and dividend variables in this equation are often referred to as the basis. From this equation, other synthetic relationships can be algebraically derived, like the synthetic long put.

[1] Note, for simplicity, simple interest is used in the computation.

Put = Call – Stock + Strike – Interest + Dividend

A synthetic long put is created by buying a call and selling (short) stock. The at-expiration diagrams in **FIGURE 6.2** show identical payouts for these two trades.

Figure 6.2 Long Put vs. Long Call + Short Stock



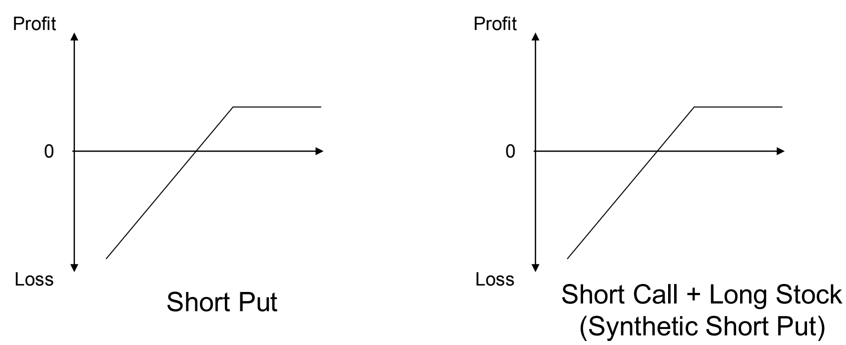
The concept of synthetics can become more approachable when studied from the perspective of delta as well. Take the 50-strike put and call listed on a $50 stock. A general rule of thumb in the put-call pair is that the call delta plus the put delta equals 1.00 when the signs are ignored. If the 50 put in this example has a –0.45 delta, the 50 call will have a 0.55 delta. By combining the long call (0.55 delta) with short stock (–1.00 delta), we get a synthetic long put with a –0.45 delta, just like the actual put. The directional risk is the same for the synthetic put and the actual put.

A synthetic short put can be created by selling a call of the same month and strike and buying stock on a share-for-share basis. This is indicated mathematically by multiplying both sides of the put-call parity equation by –1:

–Put = –Call + Stock –Strike + Interest –Dividend

The at-expiration diagrams, shown in **FIGURE 6.3**, are again conceptually the same.

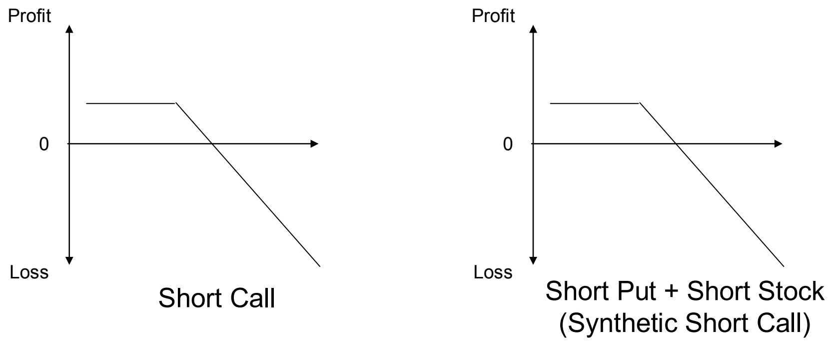
Figure 6.3 Short Put vs. Short Call + Long Stock



A short (negative) put is equal to a short (negative) call plus long stock, after the basis adjustment. Consider that if the put is sold instead of buying stock and selling a call, the interest that would otherwise be paid on the cost of the stock up to the strike-price is a savings to the put seller. To balance the equation, the interest benefit of the short put must be added to the call side (or subtracted from the put side). It is the same with dividends. The dividend benefit of owning the stock must be subtracted from the call side to make it equal to the short put side (or added to the put side to make it equal the call side).

The same delta concept applies here. The short 50-strike put in our example would have a 0.45 delta. The short call would have a –0.55 delta. Buying one hundred shares along with selling the call gives the synthetic short put a net delta of 0.45 (–0.55 plus 1.00).

Figure 6.4 Short Call vs. Short Put + Short Stock



Similarly, a synthetic short call can be created by selling a put and selling (short) one hundred shares of stock. **FIGURE 6.4** shows a conceptual overview of these two positions at expiration.

Put-call parity can be manipulated as shown here to illustrate the composition of the synthetic short call.

–Call = –Put –Stock + Strike –Interest + Dividend

Most professional traders earn a short stock rebate on the proceeds they receive when they short stock—an advantage to the short-put–short-stock side of the equation. Additionally, short-stock sellers must pay dividends on the shares they are short—a liability to the married-put seller. To make all things equal, one subtracts interest and adds dividends to the put side of the equation.

Comparing Synthetic Calls and Puts

The common thread among the synthetic positions explained above is that, for a put-call pair, long options have synthetic equivalents involving long options, and short options have synthetic equivalents involving short options. After accounting for the basis, the four basic synthetic option positions are:

Long Call = **Long** Put + Long Stock
Short Call = **Short** Put + Short Stock
Long Put = **Long** Call + Short Stock
Short Put = **Short** Call + Long Stock

Because a call or put position is interchangeable with its synthetic position, an efficient market will ensure that the implied volatility is closely related for both. For example, if a long call has an IV of 25 percent, the corresponding put should have an IV of about 25 percent, because the long put can easily be converted to a synthetic long call and vice versa. The greeks will be similar for synthetically identical positions, too. The long options and their synthetic equivalents will have positive gamma and vega with negative theta. The short options and their synthetics will have negative gamma and vega with positive theta.

American Exercise Options

Put-call parity was designed for European-style options. The early exercise possibility of American-style options gums up the works a bit. Because a call (put) and a synthetic call (put) are functionally the same, it is logical to assume that the implied volatility and the greeks for both will be the same, too. This is not necessarily true with American-style options. However, put-call parity may still be useful with American options when the limitations of the equation are understood. With at-the-money American-exercise options, the differences in the greeks for a put-call pair are subtle. **FIGURE 6.5** is a comparison of the greeks for the 50-strike call and the 50-strike put with the underlying at $50 and sixty-six days until expiration.

Figure 6.5 Greeks for a 50-Strike Put-Call Pair on a $50 Stock

	Call	Put
Delta	.554	.457
Gamma	.075	.078
Theta	.020	.013
Vega	.084	.084

The examples used earlier in this chapter in describing the deltas of synthetics were predicated on the rule of thumb that the absolute values of call and put deltas add up to 1.00. To be a bit more realistic, consider that because of American exercise, the absolute delta values of put-call pairs don't always add up to 1.00. In fact, Figure 6.5 shows that the call has closer to a –.554 delta. The put struck at the same price then has a .457 delta. By selling one hundred shares against the long call, we can create a combined-position delta (call delta plus stock delta) that is very close to the put's delta. The delta of this synthetic put is –0.446 (0.554 minus 1.00). The delta of a put will always be similar to the delta of its corresponding synthetic put. This is also true with call–synthetic-call deltas. This relationship mathematically is

$$\Delta \text{ put} \approx \Delta \text{ synthetic put}$$
$$\Delta \text{ call} \approx \Delta \text{ synthetic call}$$

This holds true whether the options are in-, at-, or out-of-the-money. For example, with a stock at $54, the 50-put would have a –.205 delta and the call would have a .799

delta. Selling one hundred shares against the call to create the synthetic put yields a net delta of –.201.

$$-.205 \approx -.201$$

If long or short stock is added to a call or put to create a synthetic, delta will be the only greek affected. With that in mind, note the other greeks displayed in Figure 6.5—especially theta. Proportionally, the biggest difference in the table is in theta. The disparity is due in part to interest. When the effects of the interest component outweigh the effects of the dividend, the time value of the call can be higher than the time value of the put. Because the call must lose more premium than the put by expiration, the theta of the call must be higher than the theta of the put.

American exercise can also cause the option prices in put-call parity to not add up. Deep in-the-money puts can trade at parity while the corresponding call still has time value. The put-call equation can be unbalanced. The same applies to calls on dividend-paying stocks as the dividend date approaches. When the date is imminent, calls can trade close to parity while the puts still have time value. The role of dividends will be discussed further in Chapter 8.

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